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MATDIP301

Third Semester B.E. Degree Examination, June/July 2011
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Express $\frac{(1+i)(2+i)}{3+i}$ in the form a + i b. **(05 Marks)**

b. Put the complex number $1-i\sqrt{3}$ in polar form. **(05 Marks)**

c. Simplify $\frac{(\cos 6\theta - i\sin 6\theta)^3(\cos 2\theta + i\sin 2\theta)^7}{(\cos 4\theta - i\sin 4\theta)^3}$. **(05 Marks)**

d. Find the cube roots of $1 - i$. **(05 Marks)**

2 a. Find the n^{th} derivative of $e^{ax}\sin(bx + c)$. **(06 Marks)**

b. Find the n^{th} derivative of $\frac{x+3}{(x-1)(x+2)}$. **(07 Marks)**

c. If $y = e^{m\sin^{-1}x}$ then prove that $(1-x^2)y_{n+2} - (2n+1)\,xy_{n+1} - (n^2+m^2)\,y_n = 0$. **(07 Marks)**

3 a. With usual notation, prove that $\tan\phi = r\frac{d\theta}{dr}$. **(06 Marks)**

b. Show that the curves $r = a(1+\cos\theta)$ and $r = a(1-\cos\theta)$ interest orthogonally. **(07 Marks)**

c. Expand $\log(1+x)$ in ascending power's of x as for as the terms containing x^4. **(07 Marks)**

4 a. If $u = e^{ax+by} f(ax-by)$, prove that $b\frac{\partial u}{\partial x} + a\frac{\partial u}{\partial y} = 2abu$. **(06 Marks)**

b. If u is a homogenous function of degree 'n' then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$. **(07 Marks)**

c. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$. find $J\left(\frac{u,v,w}{x,y,z}\right)$. **(07 Marks)**

5 a. Obtain the reduction formula for $\int\cos^n x\,dx$ where 'n' is a positive integer and hence evaluate $\int\cos^5 x\,dx$. **(06 Marks)**

b. Evaluate $\int_0^1 x^6\sqrt{1-x^2}\,dx$. **(07 Marks)**

c. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dz\,dy\,dx$. **(07 Marks)**

6 a. Evaluate $\int_0^\infty x^{3/2} e^{-4x} dx$. (06 Marks)

b. Prove that $\beta(m,n) = \frac{\lceil(m) \cdot \lceil(n)}{\lceil(m+n)}$. (07 Marks)

c. Prove that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}} d\theta = \pi$. (07 Marks)

7 a. Solve $\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$. (06 Marks)

b. Solve $\frac{dy}{dx} = \cos(x+y+1)$. (07 Marks)

c. Solve $x^2y\, dx - (x^3 + y^3)\, dy = 0$. (07 Marks)

8 a. Solve $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$. (06 Marks)

b. Solve $(D^2 + 3D + 2)y = x^2 + 3x + 1$. (07 Marks)

c. Solve $(D^2 + 4)y = \sin^2 2x$. (07 Marks)

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